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Dependence of Hertzian fracture angle on Poisson's ratio and indentation techniques

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Sub-parallel concentric conic fractures can be produced consecutively in Perspex by "liquid indentation". There is a linear relationshp between the conic angle, θ, of Hertzian fracture and the Poisson's ratio, ν, of the material. This angle is about 10° higher than the calculated path of the principal stress trajectory for a given ν. θ decreases with the increase of stress application rate. This dependence of θ is very marked in materials with high Poisson's ratios.

1. Introduction

Indentation techniques are being used increasingly in the research of the mechanical properties of brittle solids [1]. So far, Hertizian fracture has been mostly investigated as a result of mechanical indentation by the application of elastic indenters. In addition, Hertzian fracture has been induced by a thermal technique [2] and, more recently, by liquid indentation [3] and air indentation [4]. Liquid indenters have an advantage over elastic indenters in experimenting with high Poisson's ratio materials. Hertzian fracture is often difficult to form with stiff indenters in materials with high Poisson's ratio because of plastic deformation, but can be readily reproduced with liquid or air indenters which do not cause such deformation in the indented materials. The Hertzian conic angle, θ, has been linearly correlated with the Poisson's ratio, ν, of the indented material [5]. A comparison of conic angles of materials with different Poisson's ratios is the main topic of the present study.

2. Experimental procedure

It has been shown [2] that Hertzian cones were reproducibly formed in soda-lime glass due to thermally-induced stresses without the use of an elastic indenter. It would be then justified to hypothesize that an elastic indenter is not a prerequisite for Hertzian fracture. A plastic "indenter" may produce similar results provided stresses are applied by a suitable hydraulic system.

Various procedures in the application of liquid indenters on annealed Perspex have been tested. The basic experimental set-up is shown in Fig. 1. Mercury and grease were successfuly used as liqud indenters and very well defined conic fractures were repeatedly produced by this technique.

The rate of stress application is important. If the hydraulic pressure is applied slowly the typical result is a vertical fracture parallel to the cylindrical axis (of the Perspex), due to predominant hoop stress [6, 7]. Fracture initiates from the "best" oriented, sized and shaped existing flaw on the wall of the drilled cylindrical hole. If, however, stresses are rapidly applied, stress concentration occurs preferentially at the initial ring of the conic fracture. Simultaneous vertical and conic fractures were also developed.

A double conic fracture induced by grease indentation (Fig. 1) is actually not much different from a single conic fracture induced by mercury indentation (see Fig. 4 of [3]). The sub-parallel concentric conic fractures were developed consecutively. Such indentation procedures have not been the common practice in glass experimentation.

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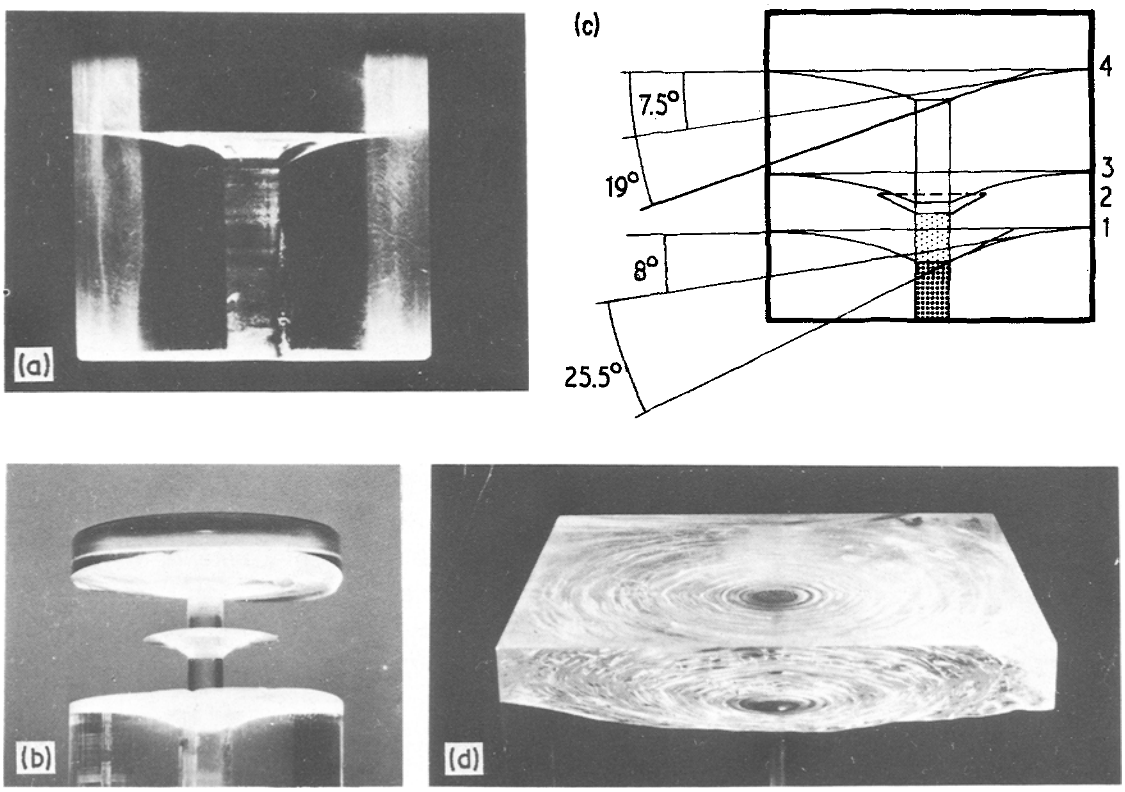

Figure 1 Multiple concentric conic fractures in Perspex. (a) The initiation of the first cone occurred at the upper end of a cylindrical hole (26 mm in length and 7.5 mm in diameter) drilled into a Perspex cylinder (42 mm in height and 48 mm in diameter), following an upward hydraulic pressure of grease applied instantaneously through the hole by a steel piston (not shown). Note that the initial conic angle is relatively low (65 ± 1°) and that it gradually increases to 83 ± 1°. Further deepening of the hole by 1.5 mm was followed by a similar "liquid-indentation" which resulted in a second symmetric, sub-parallel conic fracture. (b) The material bounded by the two conic fractures is removed to the centre of the picture. (c) Features 1, 2 and 4 are identical to (b). Fracture 3 would develop if the second deepening of the hole in (a) was longer than 1.5 mm. (d) A concentric conic fracture in Perspex produced with explosive and air indenter.

Air indentation in Perspex has also been used. A small-diameter hole is drilled in an annealed block of Perspex and an explosive (Cordtex or similar) is inserted at the collar of the hole, leaving an air-gap between the base of the hole and the explosive. The explosion causes the air to act as a piston-like indenter and produces a conical fracture whose apex is at the base of the hole. A wavy cone fracture was obtained which resembled such a feature derived by a thermal method [2] (see Fig. 1d). Indentation using air and a slow-burning source such as gunpowder gives rise to a vertical fracture that resembles similar fractures formed by liquid indentation.

3. Results and discussion

It has been repeatedly observed (Fig. 1 and additional experiments) that the conic fracture in Perspex is curved, being steeper at the beginning (initial conic angle, θ, is in the range 64.5 to 67.0°) and becomes flatter at its continuation (final θ is in the range 82 to 85°). A possible explanation to this change in fracture trend may be related to a gradual change in fracture mode. Knight *et al.* [8] have shown that the conic angle of the Hertzian fracture varies under impact conditions with the impact velocity. The implication is that when Hertzian stresses are rapidly applied a decrease in the conic angle is expected compared with slow fracture. The Hertzian fractures are being instantaneously induced in the Perspex [3]. Therefore, the initial conic angle reflects rapid Herzian fracture conditions. On further propagation the influence of the initial conditions subsides and the final conic angle gradually becomes more a function of the Poisson's ratio, ν, of the indented material [5]. In Fig. 1 the initial conic angles at the two concentric sub-parallel cones are 65 ± 1°,

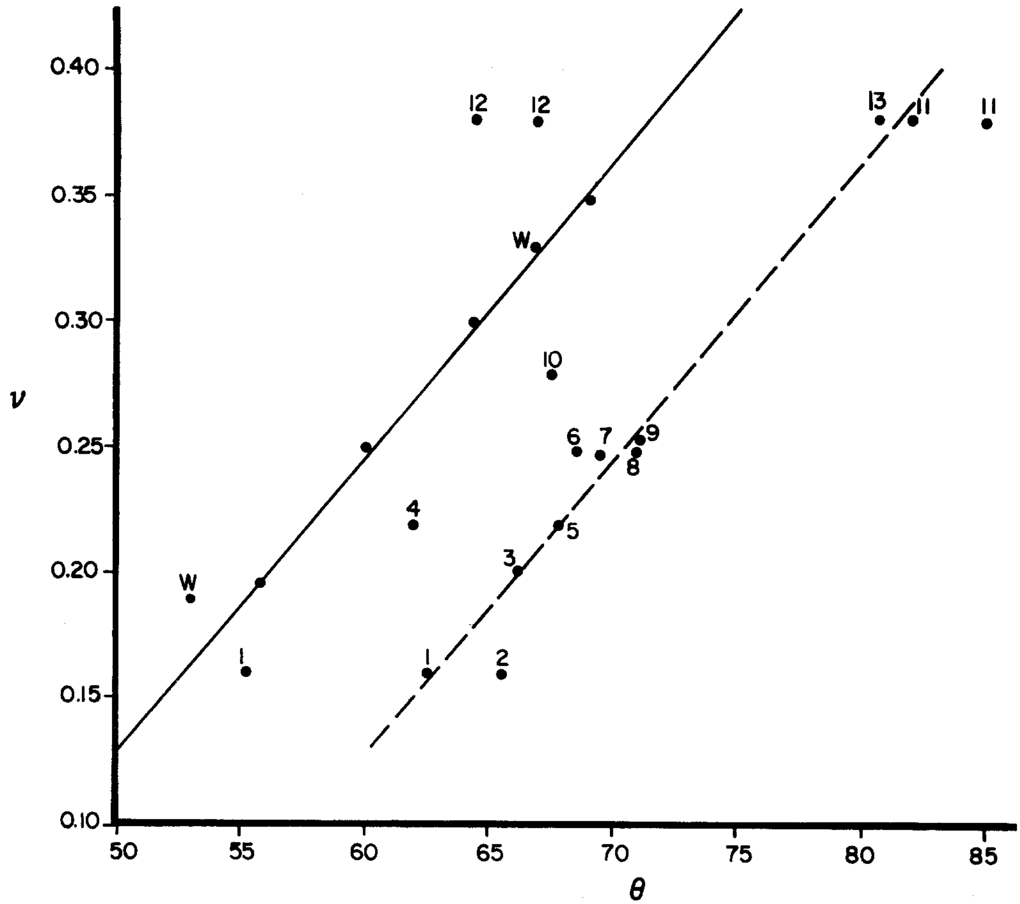

Figure 2 A plot of the Hertzian conic angle, θ, (the angle produced between the conic fracture and conic axis) against the Poisson's ratio, ν, for various materials. The solid line represents the calculated principal stress trajectory (PST) for a spherical indenter [5]; the two W points are crack paths calculated by [9]. Experimental results are: (1) chert (probably quasi-static) [10]; (2) fused silica [11]; (3) Pyrex [12]; (4) Pyrex (quasi-static) [8]; (5) Pyrex [5]; (6) soda-lime [13]; (7) soda-lime [14]; (8) soda-lime [15]; (9) soda-lime (thermal indentation) [2]; (10) vitreous carbon [16]; (11) Perspex ("liquid indentation" under static conditions); (12) Perspex ("liquid indentation" under quasi-static conditions); (13) Perspex ("air indentation"). The dashed line represents the relationship between θ and ν, for static experimental results. Geophysical methods [17] provide different values for ν; these were not used in the present study.

and the two respective final conic angles are $83 \pm 1°$. The linear variation of ν with θ is shown in Fig. 2 where the dashed line is drawn arbitrarily parellel to the calculated principal stress trajectory (PST) line based on [5]. Nevertheless, the experimental static results seem to support this linear relationship between θ and ν. However, for a given ν the experimental results for θ found in this work are higher (by about $10°$) than the calculated PST results, as observed before [5]. The conic fractures produced under quasi-static conditions have θ-values close to the calculated PST line. Hence, the predictions made by Warren [9] would be valid for quasi-static or dynamic conditions. There are two points of Sample 1 in Fig. 2. These represent scatter in results obtained for prehistoric cone fractures in chert. These fractures are interpreted on the basis of stress calculations to have been generally derived by impact by falling weights from a height of 1 to 2 metres [18] and assumed to represent various quasi-static conditions. It was mentioned that θ decreases under dynamic conditions with the increase of impact velocities [8]. Therefore, Sample 1 with $\theta = 62.5$ is expected to have been produced under conditions close to static and, as such, it falls close to Sample 2. Sample 4 shows the same trend of decrease θ value of Pyrex under quasi-static conditions. Samples 3 and 5 are both Pyrex (Corning 7740), but the respective authors consider different ν

values for these glasses. Sample 6 shows a result from soda-lime glass derived by a punch indenter. Sample 6 gives a value very similar to $21.5 \pm 2°$ derived by Phillips using a tungsten carbide sphere [19] (see Fig. 21 of [1]), and it somewhat varies from the value from Sample 8 obtained by the same indenter on a float glass. Sample 9 is a result of "thermal indentation" at the range of 170 to 230° C in an unannealed glass. The ν-value is assumed to be slightly higher than normal for soda-lime glass. Sample 10 shows a lower θ-value than expected. This may possibly be assigned to the higher density and strength [16] and perhaps stiffness of the surface layer of vitreous carbon. Sample 11 represents readings in Perspex under static conditions. Sample 12 shows much lower θ-values for Perspex under quasi-static conditions. It is implied from these results that the difference in θ-values between static and quasi-static conditions is considerably higher in materials with high ν-values (low stiffness) than for materials with low ν-values.

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